HOMEWORK 8

Do problems 12.3, 12.5 and problems 16 and 18 from the Problems III questions. Additionally, solve the following problems:

Problem 1

Let X, Y, and Z be finite, nonempty sets such that $|X| = a$, $|Y| = b$, and $|Z| = c$. State and prove a formula for the number of injections $X \to Y \times Z$.

Problem 2

Let n be a positive integer and $1 \leq k \leq n$. Prove that $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$.

Problem 3

Let n be a positive integer. Give a bijection

$$f : \bigcup_{k=0}^{n} \mathcal{P}_k(\{1, 2, \ldots, n\}) \times \mathcal{P}_{n-k}(\{n+1, n+2, \ldots, 2n\}) \to \mathcal{P}_n(\{1, 2, \ldots, 2n\}).$$

Be sure to carefully define f, and prove that it is indeed a bijection. Use this to prove

$$\sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \binom{2n}{n}.$$